A numerical study of flame instability and cell dynamics of opposed nonpremixed tubular flames with radiative heat loss

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Abstract

The flame instability and cell dynamics of opposed nonpremixed tubular flames with radiative heat loss are investigated using high fidelity numerical simulations with linear stability analysis. Two different extinction Damköhler numbers (i.e. large radiation-induced extinction Damköhler number, \(DA_{RF,E}\) and high stretch-induced extinction Damköhler number, \(DA_{RF,E}\)) are obtained by 1-D analysis. It is verified by 2-D simulations using the perturbed initial condition (IC) that the flammable region near \(DA_{RF,E}\) is divided by three critical \(Da\) from the linear stability analysis: \(Da_{c,1}\) denotes \(Da\) at which first cellular instability occurs; \(Da_{c,2}\) represents the beginning of growing oscillation; \(Da_{c,3}\) indicates the unsteady radiative extinction \(Da\) at which flame cannot survive due to too strong oscillation although the corresponding 1-D solutions exist. At extremely large 1-D, the extinction of two propagating flame cells by a head-on quenching and a rotating flame are found by using different shapes of IC.

Introduction

Limit phenomena of nonpremixed flames have been extensively investigated by experiments and numerical simulations with various configurations such as counter flow, axisymmetric jet and tubular flames [1–6]. Especially, the tubular configuration has an advantage that the stretch rate and curvature are independent from each other, and hence, it has been widely used to investigate their effects on flames [1–4]. Recently, the diffusive-thermal instability of nonpremixed flames was investigated by using 1-D/2-D simulations with the linear stability analysis [6]. The accuracy of prediction from stability analysis and the reasons of the discrepancy between experiments and numerical simulations were elucidated [6]. Although there are many studies on the high stretch-induced extinction, the effects of radiation at large Damköhler number, \(Da\), have rarely been studied.

The existence of two extinction limits is already known by the numerical analysis. Therefore, there have been many efforts to uncover the phenomena near the radiative extinction. In a stagnant mixing layer with unity Lewis number, three types of flame evolution were identified near large \(Da\) region: the decaying oscillatory solution, diverging solution to extinction, and stable limit-cycle solution [7]. When the Lewis number of fuel is less than the unity, the structure and dynamic response of radiative counter flow flames were investigated by 1-D/2-D simulations using various initial profiles. The stationary cellular structure, wavy flames, and propagating cellular flames were found as different \(Da\) [8].

In the present study, the instability characteristics of nonpremixed tubular flame near the large radiation-induced extinction are studied by 1-D/2-D simulations with the linear stability analysis. The cell dynamics with different initial conditions are also elucidated.

Problem Formulation

To find two extinction \(Da\), 1-D analysis is conducted. The normalized 1-D governing equations of the temperature, \(T\), and the mass fraction of fuel and oxidizer, \(Y_F\) and \(Y_O\), are given by:

\[
\begin{align*}
\frac{u_r}{r} \frac{d}{dr} \left( \frac{T}{Y_F} \right) &= \frac{1}{r} \frac{d}{dr} \left( \frac{1}{Y_O} \right) \left( \frac{T}{Y_F/Y_O} \right) + \frac{q}{\alpha} \frac{Y_F}{\bar{Y}_{O,0.2}} - Ra \left( \frac{T}{T_\infty} - \frac{T_\infty}{4} \right) \\
Da Y_F Y_O e^{-T/a/T} &= \left( \frac{q}{\alpha} \frac{Y_F}{Y_{F,1}} - Ra \left( \frac{T}{T_\infty} - \frac{T_\infty}{4} \right) \right).
\end{align*}
\]

They are based on the adiabatic nonpremixed tubular flames analysis. The radiative heat loss term is included in the temperature equation to investigate the radiation effect on the extinction, where \(Ra\) is the constant for radiative heat loss. It can be also expressed as the product of the intensity of radiation, \(I\), and \(Da\) (=1×\(Da\)). The detail about other parameters and boundary conditions are introduced in [6].

The equation (1) is solved by the Newton-Raphson method with a simple continuation method. The maximum temperature, \(T_{max}\), as a function of \(Da\) for different \(I\), is shown in Fig. 1. Two different extinction \(Da\)s (i.e. large radiation-induced extinction Damköhler number, \(DA_{RF,E}\), and high stretch-induced extinction Damköhler number, \(DA_{RF,E}\)) are found, and the flammable region significantly decreases with increasing \(I\). The \(DA_{RF,E}\)s with various \(I\) are approximately equal to the value, 13950, in the adiabatic flame. However, it increases significantly in the extremely high \(I = 3.7 \times 10^7\). To study for general effects of radiative heat loss, \(I = 10^4\) is adopted in the present study.

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Figure 1 The maximum temperature, \( T_{\text{max}} \), as a function of Damköhler number, \( Da \), for different intensity of radiation, \( I \).

Results and Discussion

Prior to performing 2-D simulations, the linear stability analysis is conducted to predict the characteristics of nonpremixed tubular flame near \( Da_{R,E} \).

Figure 2 shows the largest growth rate, \( \lambda_R \), as a function of wavenumber, \( k \), for different Damköhler numbers, \( Da \). The sign of \( \lambda_R \) is important to understand the cellular instability. If at least one positive \( \lambda_R \) is found over the range of \( k \) at a given \( Da \), the cellular instability occurs even by infinitesimally small disturbances. In addition, the imaginary part of \( \lambda, \lambda_i \), can exist near \( Da_{R,E} \) unlike \( Da_{E} \), which implies that the oscillatory flame response can occur in time. We identified \( Da_{C1} = 5.76 \times 10^7 \) at which the first positive \( \lambda_R \) is observed near \( Da_{R,E} \), and \( Da_{C2} = 6.17 \times 10^7 \) at which the first positive \( \Re(\lambda) \) is found.

Based above the definition, the \( Da \) regimes can be divided by two critical \( Da \): Regime I (\( Da < Da_{C1} \)), II (\( Da_{C1} \leq Da < Da_{C2} \)) and III (\( Da \geq Da_{C2} \)). In Regime I, fully stable tubular flame is expected due to no cellular and growing oscillatory instability. On the other hands, the cellular instability occurs by positive \( \lambda_R \) in Regime II. Although the stable tubular flame is easy to become unstable due to growing fluctuation in Regime III, the cell formation would be promoted by growing oscillation. To identify the prediction from the linear stability analysis, 2-D simulations are carried out.

Figure 3 shows three different initial conditions (IC) for 2-D simulations. The perturbed IC (Fig. 3a) in which small disturbance exists is basically used to identify fundamental flame characteristics such as the cellular instability, oscillatory instability, and total extinction \( Da \). The simulations for observing instability characteristics are based on 1-D results of identical \( Da \) (Case 1). On the other hands, once a cellular flame at a given \( Da \) is obtained, successive simulations at successively larger \( Da \) are reproduced from each of the previous steady solutions, resetting the simulation time to the beginning upon restarting to find total extinction \( Da \) (Case 2).

The effects of symmetric and asymmetric large disturbance with high radiative heat loss are confirmed by the small C-shaped and asymmetric IC (Fig. 3b and c). To induce strong radiative heat loss, 2-D simulations using the 1-D solution at \( Da_{C2} = 6.17 \times 10^7 \) are conducted at extremely large \( Da \) beyond \( Da_{R,E} \) (Case 3 and 4).

Figure 3 Three different initial conditions for 2-D simulations: (a) the perturbed IC, (b) the small C-shaped IC, and (c) the asymmetric IC.
The results of 2-D simulations using perturbed IC (Case 1) and the linear stability analysis are shown in Table 1. The wavenumber corresponding to the maximum $\lambda_R$ is denoted by $k_{\text{max}}$, which is related to the number of flame cells, $N_{\text{cell}}$. Several points are noted from the table. First, the cellular instability near $Da_{R,E}$ occurs in the range of $Da_{C,1}$ to $Da_{C,2}$ as predicted from the linear stability analysis. Beyond $Da_{C,3}$ ($\approx 6.25 \times 10^7$), however, although 1-D stable solutions exist, the corresponding 2-D turbulent flame is extinguished by strong radiative heat loss.

Second, the $k_{\text{max}}$ slightly decreases with increasing $Da$, and $N_{\text{cell}}$ is approximately similar to $k_{\text{max}}$: $k_{\text{max}} = 28.6 \rightarrow 27.5$, and $N_{\text{cell}} = 29 \rightarrow 26$. However, some cases at relatively far from $Da_{C,1}$ show a lack of the accuracy, because the neighbour wavenumbers of $k_{\text{max}}$ is large enough to affect cell formation.

Third, the oscillation characteristics like damped and growing can be distinguished by $Da_{C,2}$. To investigate the characteristics of the oscillation in detail, the temperature change of nonpremixed tubular flames in the time for different $Da$ is shown in Fig. 4. As mentioned above, the damped cases whether noncellular or cellular flames are found in Regime I and II as shown in Fig. 4a. In Regime III, growing and cellular flames, represents in Fig. 4b. Unlike the linear stability analysis, the extinction by the growing oscillation is observed in Regime IV ($Da \geq Da_{C,3}$) as shown in Fig. 4c. Even though, the linear stability analysis cannot identify Regime IV, it can be concluded that the linear stability analysis may predict the instability characteristics such as cellular and oscillatory instability with a good accuracy.

Apart from this, the total extinction $Da$ ($= 1.7 \times 10^6$) is found beyond 1-D extinction $Da_{R,E}$ (Case 2). The size of each cell decreases as approaching to the extinction likewise adiabatic flames. $T_{\text{max}}$ does not change in a wide range due to strong reaction, though the radiative heat loss is also large.

To investigate the effects of symmetric and asymmetric large disturbance, the 2-D simulations are performed at large $Da$ over $Da_{R,E}$ (Case 3 and 4). Figure 5 shows the temperature isocontours using the small C-shaped IC at $Da = 5.0 \times 10^7$. Under large radiative heat loss conditions, the initial flame splits in two flame cells propagating towards the opposite direction and finally flame extinction appears when two flame cells collide each other.

Figure 6 shows the temperature isocontours using the asymmetric IC at $Da = 5.0 \times 10^7$. Unlike the previous result, the intensity of initial two flame cells is different from each other due to asymmetry of disturbance. Therefore, the relatively weak flame cell

![Table 1: Summary of the results using Method 1 as a function of Damköhler number, $Da$, with the linear stability analysis. E, D, and G represent extinction, damped, and growing, respectively.](image)

<table>
<thead>
<tr>
<th>$Da$ ($\times 10^7$)</th>
<th>$k_{\text{max}}$</th>
<th>$N_{\text{cell}}$</th>
<th>Oscillation</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.7 ($\lambda_R &lt; 0$)</td>
<td>29.0</td>
<td>0</td>
<td>D</td>
</tr>
<tr>
<td>5.8 ($\lambda_R &gt; 0$)</td>
<td>28.6</td>
<td>29</td>
<td>D</td>
</tr>
<tr>
<td>5.9</td>
<td>28.3</td>
<td>29</td>
<td>D</td>
</tr>
<tr>
<td>6.0</td>
<td>28.0</td>
<td>29</td>
<td>D</td>
</tr>
<tr>
<td>6.1</td>
<td>27.7</td>
<td>27</td>
<td>D</td>
</tr>
<tr>
<td>6.17 ($\lambda_R = 0$)</td>
<td>27.6</td>
<td>27</td>
<td>G</td>
</tr>
<tr>
<td>6.2</td>
<td>27.5</td>
<td>26</td>
<td>G</td>
</tr>
<tr>
<td>6.3</td>
<td>27.2</td>
<td>E</td>
<td>G</td>
</tr>
<tr>
<td>6.5</td>
<td>26.7</td>
<td>E</td>
<td>G</td>
</tr>
</tbody>
</table>

![Figure 4](image) The maximum temperature, $T_{\text{max}}$, change with time for different Damköhler numbers, $Da$, in 2-D simulations: (a) the damped oscillation, (b) the growing oscillation with cellular instability, and (c) the extinction.

![Figure 5](image) Temperature isocontours using the small C-shaped IC with time at $Da = 5.0 \times 10^7$: Collision and extinction.
Figure 6 Temperature isocontours using the asymmetric IC with time at $Da = 5.0 \times 10^7$: Rotating.

Figure 7 Summary of numerical simulations with linear stability analysis near large radiation-induced extinction Damköhler number, $Da_{R,E}$.

do not propagate and disappears while the other survives and rotates continuously.

Conclusions

The characteristics of nonpremixed tubular flames with radiative heat loss are investigated by 1-D/2-D numerical simulations, and the results from 1-D/2-D simulations are included in Fig. 7a. Near the radiation-induced extinction $Da$, the instability information such as the critical $Da$ and $N_{cell}$ can be obtained by the linear stability analysis with a good accuracy, and these are verified by the comparison with 2-D simulations. The vicinity of $Da_{R,E}$ is divided into four regimes by three critical $Da$ and are arranged in Fig. 7b. Since the diffusive-thermal instability, which makes the cell formation to be helpful for survival in the harsh condition, occurs, the extinction limit is extended beyond $Da_{R,E}$. The interesting phenomena, collision and a rotating flame, which cannot be seen near high stretch-induced extinction limit are observed when the reaction and heat loss are balanced in the extreme conditions.

Acknowledgements

This work was supported by the Space Core Technology Development Program (No.2015M1A3A3A02027319) and Basic Science Research Program (No.2015R1A2A2A01007378) through the National Research Foundation of Korea grant funded by the Ministry of Science, ICT and Future Planning.

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